1 A rock of mass 8 kg is acted on by just the two forces $-80\mathbf{k}$ N and $(-\mathbf{i} + 16\mathbf{j} + 72\mathbf{k})$ N, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane and $\mathbf{k}$ is a unit vector vertically upward.

(i) Show that the acceleration of the rock is $\left(\ \tfrac{1}{8}\mathbf{i} + 2\mathbf{j}\ \ \mathbf{k}\right)\mathrm{ms}^{-2}$. [2]

The rock passes through the origin of position vectors, O, with velocity $(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$ m s ^{1} and 4 seconds later passes through the point A.

(ii) Find the position vector of A. [3]

(iii) Find the distance OA. [1]

(iv) Find the angle that OA makes with the horizontal. [2]

2 Fig. 4 shows the unit vectors $\mathbf{i}$ and $\mathbf{j}$ in the directions of the cartesian axes Ox and Oy, respectively. O is the origin of the axes and of position vectors.

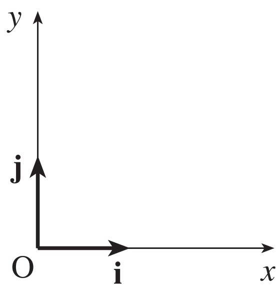

Fig. 4

The position vector of a particle is given by $\mathbf{r} = 3t\mathbf{i} + (18t^2 - 1)\mathbf{j}$ for $t \geqslant 0$, where t is time.

(i) Show that the path of the particle cuts the x-axis just once. [2]

(ii) Find an expression for the velocity of the particle at time t.

Deduce that the particle never travels in the $\mathbf{j}$ direction. [3]

(iii) Find the cartesian equation of the path of the particle, simplifying your answer. [3]

3 In this question, the unit vectors $\begin{pmatrix} \\ \end{pmatrix}$ and $\begin{pmatrix} \\ \end{pmatrix}$ are in the directions east and north.

Distance is measured in metres and time, t, in seconds.

A radio-controlled toy car moves on a flat horizontal surface. A child is standing at the origin and controlling the car.

When $t = 0$, the displacement of the car from the origin is $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ m, and the car has velocity $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ m s^{-1}.

The acceleration of the car is constant and is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ m s^{-2}.

(i) Find the velocity of the car at time t and its speed when $t = 8$. **[4]**

(ii) Find the distance of the car from the child when $t = 8$. **[4]**

4 At time t seconds, a particle has position with respect to an origin O given by the vector

$$\mathbf{r} = \begin{pmatrix} 8t \\ 10t^2 - 2t^3 \end{pmatrix},$$

where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are perpendicular unit vectors east and north respectively and distances are in metres.

(i) When $t = 1$, the particle is at P. Find the bearing of P from O. **[2]**

(ii) Find the velocity of the particle at time t and show that it is never zero. **[3]**

(iii) Determine the time(s), if any, when the acceleration of the particle is zero. **[3]**

5 A particle of mass 5 kg has constant acceleration. Initially, the particle is at $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ m with velocity $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ m s^{-1}; after 4 seconds the particle has velocity $\begin{pmatrix} 12 \\ 9 \end{pmatrix}$ m s^{-1}.

(i) Calculate the acceleration of the particle. **[2]**

(ii) Calculate the position of the particle at the end of the 4 seconds. **[3]**

(iii) Calculate the force acting on the particle. **[2]**

6 A toy boat moves in a horizontal plane with position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors east and north respectively. The origin of the position vectors is at O. The displacements x and y are in metres.

First consider only the motion of the boat parallel to the x-axis. For this motion

$$x = 8t - 2t^2.$$

The velocity of the boat in the x-direction is v_x m s^{-1}.

(i) Find an expression in terms of t for v_x and determine when the boat instantaneously has zero speed in the x-direction. **[3]**

Now consider only the motion of the boat parallel to the y-axis. For this motion

$$v_y = (t - 2)(3t - 2),$$

where v_y m s^{-1} is the velocity of the boat in the y-direction at time t seconds.

(ii) Given that $y = 3$ when $t = 1$, use integration to show that $y = t^3 - 4t^2 + 4t + 2$. **[4]**

The position vector of the boat is given in terms of t by $\mathbf{r} = (8t - 2t^2)\mathbf{i} + (t^3 - 4t^2 + 4t + 2)\mathbf{j}$.

(iii) Find the time(s) when the boat is due north of O and also the distance of the boat from O at any such times. **[4]**

(iv) Find the time(s) when the boat is instantaneously at rest. Find the distance of the boat from O at any such times. **[5]**

(v) Plot a graph of the path of the boat for $0 \leqslant t \leqslant 2$. **[3]**